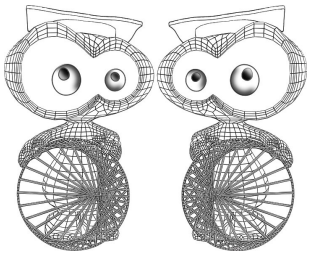


Cesenatico 2010

11th ITALIAN TEAM COMPETITION

Semifinal B – 7 Maggio 2010

General rules

- Each problem requires an answer which is an integer number between 0000 e 9999. The answer must be marked on a card.
- If the answer is not an integer, unless otherwise stated, mark its rounding to the nearest integer.
- If the answer is not a negative integer, or if the problem has no solution, mark 0000.
- If the answer is an integer greater than 9999, mark the last four digits.
- it may be useful to know the following approximations:

$$\sqrt{2} = 1.4142 \qquad \sqrt{3} = 1.7321 \qquad \sqrt{5} = 2.2361 \qquad \sqrt{7} = 2.6458 \qquad \pi = 3.1416.$$

Important deadlines

- **10 minutes from the start**: deadline to choose the special problem (after that, the first problem will be the special one).
- **30 minutes from the start**: deadline to ask questions about the text.
- **90 minutes from the start**: end of the competition.

1. Rules of engagement

The most prestigious math competition of the π-rates of the Caribbean is the Big Whoop Cup, which will be held this year on May 7th in the Year of Our Lord 1726. Captain Jack Arrow, with the help of his good friend Master Gibbs, arrives at Tortuga to recruit a crew for his ship, the True Pearl. When the nth candidate drunkenly staggers in, Jack asks him, "If n is the answer to this question, what is $\frac{(n-1)(n-3)+6}{2}$?" What should the π-rate answer to get the job?

2. A valuable scroll

Captain Arrow has obtained a treasure map, on which three points A, B, C are marked. These three points are the vertices of an isosceles triangle. The base AB is 4.4 km long, and the angle which corresponds to C measures 120 degrees. Captain Arrow stands on the point B, and he knows that the treasure is buried in the intersection of the line segment BC and the bisector of the angle in A. How many metres will he have to walk to reach the treasure?

3. The reckoning

The crew of the True Pearl is divided in two factions: the π-rates recruited by Captain Jack Arrow, who always tell the truth, and those recruited by the feared Bourbakossa, who always lie, and are planning to mutiny. All 7776 members of the crew stand in a very large circle on the bridge. Each member states, "Of the two π-rates standing next to me, one was recruited by Jack, and the other by Bourbakossa." Master Gibbs, who was the first to be recruited by Jack, is one of those standing in the circle. How many π-rates are *not* planning to mutiny?

4. Set sail for treasure

Tom^2 has a compass that always points towards the treasure, and displays the estimated time of arrival, assuming that the ship travels directly towards the treasure at 12 knots (a knot is 1 mile per hour). The ship he's sailing on, the Lined Dutchman is heading straight for the treasure at its maximum speed. Knowing that the time shown on the compass decreases by 1 minute every 8 miles, what is the speed of the ship in knots?

[As the answer, provide the *product* of the numerator and the denominator of the fraction in its simplest form.]

5. Flag to the wind

Unfortunately, the mutiny was successful, Jack Arrow was left stranded on a deserted island, and the True Pearl is now in the hands of Bourbakossa. The ship now flies his flag, which is a large black triangle on which three π-rates symbols are drawn many times. In the first row there is a monkey. In the second row, there are a monkey and a chest. In the third row, there are a monkey, a chest, and an apple. In the fourth row there are a monkey, a chest, an apple, and another monkey, and so on. On the nth row there are n symbols repeating the order of monkey, chest, and apple. The flag has 2010 rows. What is the difference between the number of monkeys and the number of apples?

6. The Tortuga Triangle

On the island of Tortuga there is a commemorative plate called the Tortuga Triangle. It is in appearance identical to Pascal's triangle, but each row is dedicated to a year between 1600 and 1726, and the numbers on each row represent the number of π-rates sailing on each of the ships that sank that year. For example, in 1600 (1) only one ship sank with one π-rate onboard. In 1601 (1, 1), two ships sank with a crew of one π-rate each. In 1602 (1, 2, 1) three ships sank with a crew of on, two, and one respectively, and so on. Master Gibbs, who is superstitious, explains that sunken ships become ghost ships if and only if they have an odd number of crew members. How many ghost ships appeared during that period of time?

7. The Aztec chest

The chest which once contained the Aztec gold is now empty, and all the coins must be recovered before the curse can be dispelled. The ghost π-rate Bourbakossa has unleashed his men to search all corners of the Caribbean, while he tries to remember how many coins were originally in the chest. The only clue is a note which Captain Jack Arrow scribbled on his log, which states, "…the Aztec coins are equal in number to the least positive integer $a = n^5 + 4n^4$ such that n is a natural number and a is the square of an odd number." How many coins will the π-rates have to find?

8. Optimiser!

When the beautiful Elizabeth Summ is captured by Bourbakossa, she invokes the "optimiser" in order to be spared, but to her dismay she finds that for the π-rates to acknowledge it, she must find the greatest positive integer a such that $a^2 + 200a + 1$ is a square number. What should she answer?

9. A difficult jailbreak

With Will's help, Jack Arrow broke out of his cell on the lowest floor of the jail tower. He wants to reach the highest floor, from which he hopes to escape. The tower has 222 floors, numbered from 1 to 222, that are connected by a number of staircases. From the 1st floor to the 2nd floor there is only one flight of stairs of only 1 step. From the 2nd floor to the 3rd floor there are two flights of stairs of 1 and 2 steps respectively. From the 3rd floor to the 4th floor there are 3 flights of stairs of 1, 2, and 3 steps, and so on. What is the average number of steps between two floors?

10. A macabre division

The π-rates of the True Pearl have found a great booty of no more than 100000 doubloons. Jack discovers that he can share the booty equally between the π-rates in his crew. Suddenly, Master Gibbs arrives and tells him, "Captain, we have lost a man." Jack grieves for a moment, then he shrugs and smiles, and informs the π-rates that it is still possible to divide the booty between the men, and that each man will receive exactly 13 extra doubloons. What is the maximum number of doubloons that each member of the crew could receive?

11. Do you fear Davy Jensen's bingo?

The cursed π-rate Davy Jensen has a bag which contains 90 white chips, and 1 blood red chip. Each prisoner on his ship, the Lined Dutchman, must play the following game: he draws the chips randomly from the bag one by one, arranging them in a triangle (the first row has 1, the second has 2, the third has 3, and so on up to 13). The unfortunate prisoner must then serve on the ship for a number of years equal to the number of rows of the triangle which are longer than the row in which the red tile appears. "Bootstrap Bill" Turing, watching from a corner, wonders what the average number of years that the prisoners will serve. Jack doesn't understand what his fellow π-rate means. Bill's son Will, who has an education, explains that there are many ways to find the average number of years of service (all of which produce the same result), but the most common is to find the sum of all the numbers $k \cdot p_k$ where, for each natural number $0 \leq k \leq 12$, p_k is the probability that the years of service will be k. What is the average number of years of service?
[As the answer, provide the *product* of the numerator and the denominator of the fraction in its simplest form.]

12. The capture of Cauchypso

The Brotherhood of π-rates is formed by 9 noble π-rates and Teague Arrow, Jack's father. Each of the members has a card from a deck of 10 magic tarot cards, numbered from 0 to 9. To capture Cauchypso in human form, the π-rates must arrange the cards to form two five-digit numbers (which can't begin with 0), such that one is twice the other. These two numbers must be the smallest that satisfy this condition. What are the last 4 digits of the smallest of the two numbers?

13. A greedy buccaneer

The swords forged by Will Turing are famous for their quality, and are often sold for large amounts of money. Will wants to finance the next expedition by selling swords at Tortuga, but he must decide how many to make, and at what price to sell them. If he sells them for 70 doubloons or less, he's certain that each of the 10000 π-rates in Tortuga will want one. If he sets a higher price, for each doubloon he adds to the price he will lose 50 customers. If it costs 10 doubloons to make a sword, and Will sets the price which will earn him the most money, how many π-rates in Tortuga will buy a sword from Will?

14. An ingenious route

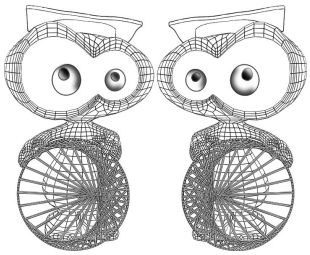
The True Pearl is hunted by the Cramer, a terrible sea monster unleashed by Davy Jensen. She is trying to reach land before the monster. The closest island is exactly 10 miles north of the ship. To take advantage of the wind which is not from the stern, Captain Arrow sets a route that alternates between two directions: travelling north-east (at a 30 degrees angle with respect to the north) at 5 m/s, and travelling north-west (at a 60 degrees angle with respect to the north) at 3 m/s. Meanwhile, the Cramer starts at a point 10 miles to the south of the ship, and swims north at a constant speed. What is the highest speed at which the Cramer can travel that allows the True Pearl to reach land first? Answer in cm/s.

15. The phantom chest

It's not easy to open the chest which contains Davy Jensen's heart. Brave Will Turing has the key, but he is dismayed to find that the chest has 4 locks, one for each side. His key fits in every lock, but to activate the mechanism to open the chest he must give the key two turns in each lock, in a specific order which he doesn't know. All he knows is that the first turn of the key (but not the second) should go in the front lock. How many times will Will have to try to activate the mechanism, at most, to open the chest?

16. Battle on the open sea

The True Pearl is in danger of being swallowed by a whirlpool! The whirlpool has the shape of a quadrilateral $OABC$, which is contained in a semicircle with centre O and diameter AD. B lies on the semi-circumference, and the angle BOA measures 60 degrees. The Lined Dutchman is fighting from a point E on the semi-circumference such that EOD measures 30 degrees. The point C lies either on the line segment DE or on the arc EB; unfortunately, the lookout can't quite tell. Knowing that the line segment AD is 48 m long, what is the greatest area, in square metres, that the whirlpool can have?



Cesenatico 2010

11th ITALIAN TEAM COMPETITION

Semifinal B – Solutions – 7 Maggio 2010

Nr.	Problem	Solution
1	Rules of engagement	**0003**
2	A valuable scroll	**1610**
3	The reckoning	**5184**
4	Set sail for treasure	**2080**
5	Flag to the wind	**1340**
6	The Tortuga Triangle	**2059**
7	The Aztec chest	**5625**
8	Optimiser!	**4900**
9	A difficult jailbreak	**8251**
10	A macabre division	**1144**
11	Do you fear Davy Jensen's bingo?	**0004**
12	The capture of Cauchypso	**3485**
13	A greedy buccaneer	**6500**
14	An ingenious route	**0588**
15	The phantom chest	**0540**
16	Battle on the open sea	**0537**