1. Let P be an internal point of the equilateral triangle ABC. The feet of the perpendiculars from P to the sides AB, BC and CA are C_1, A_1 and B_1, respectively. Given that $AC_1 = 4$, $C_1B = 8$ and $BA_1 = 5$ find the value of $CB_1 \cdot B_1A$. 20 points

2. Find the number of those 8-digit integers that decrease to one ninth of their value when their first digit is omitted. 20 points

3. There are 21 people in a queue, their respective heights are all distinct. The third smallest one among them is Andy. Starting from the front each person counts how many people taller than him are preceding him in the queue. Here is the list: 0, 0, 1, 1, 2, 2, 3, 3, … 9, 9, 10.
How many people are there taller than Andy behind him in this queue? 20 points

4. Given the triangle ABC, A_1 and B_1 are internal points of the sides BC and AC. The segments AA_1 and BB_1 intersect at M. The areas of the triangles AMB_1, AMB and BMA_1 are 3, 7 and 7 units respectively. What is the area of quadrilateral CB_1MA_1? 25 points

5.

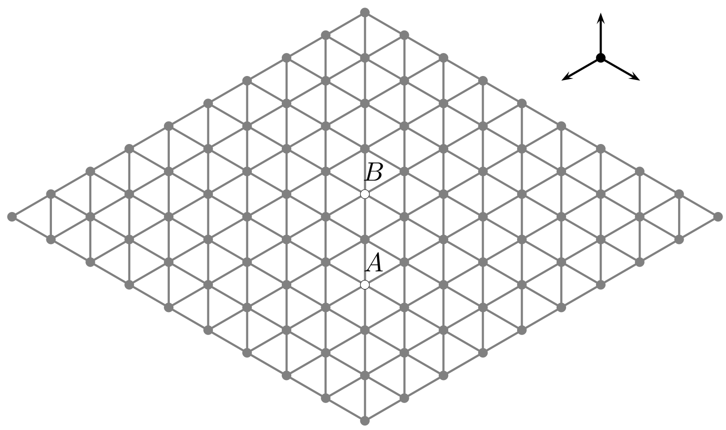
$$\prod_{k=2}^{100} \frac{k^3+1}{k^3-1} = \frac{p}{q}$$

Find the value of $p+q$ given that p and q are relatively prime positive integers. 25 points

6. The points B_1, B_2, B_3 and B_4 are on the sides of the unit square $A_1A_2A_3A_4$ such that B_i is on the side A_iA_{i+1} (where clearly $A_5 = A_1$) and $A_iB_i = \frac{1}{n}$. Find the smallest positive integer n such that the area of the square bounded by the lines A_1B_2, A_2B_3, A_3B_4 and A_4B_1 is at least 0.9. 25 points

7. The set M of positive integers has the following two properties: *i*) none of the elements of M is divisible by 7; *ii*) among any 4 elements of M there are some whose sum is divisible by 7.
What is the maximum number of the elements of the set M? 30 points

8. A point is wandering on the vertices of an *infinite* regular triangular lattice. From a given vertex it can step to an adjacent one in any of the three directions indicated in the diagram. Starting from vertex A how many ways are there to arrive to vertex B by no more than 13 steps? (Those routes that are passing B before finally arriving here are also included.) 30 points



9. Find the value of $\lfloor 100xy \rfloor$ if x and y are rational numbers satisfying

$$\sqrt{2\sqrt{3}-3} = \sqrt{x\sqrt{3}} - \sqrt{y\sqrt{3}}.$$

30 points

10. How many permutations $i_1, i_2, \ldots i_{16}$ of the numbers $1, 2, \ldots, 16$ have the following property: $|i_k - k| \le 1$ for every $1 \le k \le 16$? 30 points

11. Given is a regular tetrahedron consider those planes that are passing through an edge and the midpoint of the opposite edge. Into how many parts is the tetrahedron divided by these planes? 30 points

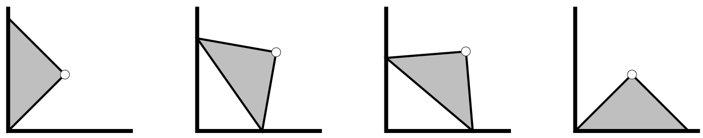
12. The lengths, in meters, of the edges of a rectangular cuboid and that of the space diagonal are integer numbers, respectively. We are also given that the measure of the surface area in m^2 is equal to that of the volume in m^3. Find, in meters, the greatest possible length of the space diagonal. 30 points

13. Let $a = 1 + \sqrt{5}$. What is the value of

$$S = (4 - a) \cdot \sqrt{2 + a} \cdot \sqrt[3]{a} \cdot \sqrt[6]{3a + 4}?$$

35 points

14. An isosceles right triangle is sliding in a rectangular region R while the endpoints of its hypotenuse are attached to the rays that form R. (See the diagram.) The lenght of the hypotenuse is 2 meter. Find the distance travelled by the-right angled vertex in centimeters. (Note: give the total "odometer distance", not the displacement or the arc length of the curve traced by the vertex.) 35 points



15. How many ways are there to select four vertices of a convex 24-gon in such a way that each side of the convex quadrilateral defined by the selected vertices is a *diagonal* of the 24-gon? (The order of the selection is not important.) 35 points

16. There are n closed boxes in a row. Each is either empty or contains a gift. You have to decide whether there are two adjacent non-empty boxes. To do this, you may open any boxes of your choice, one at a time, until you know the answer. For a given n, this task is called *difficult* if no matter how you proceed, it is possible that you have to open every box to be sure. For how many values of $n \in \{1, 2, \ldots, 2013\}$ is the task difficult? 40 points

17. For an integer $n < 10000$ the positive integer k is said to be *attached* to n if the remainder when n is divided by $2k + 1$ is equal to k. Find the value of n with the largest number of attachments. 40 points

18. The distance of points A and B is 900 centimeters. An ant wants to arrive from point A to point B along a polygonal chain consisting of 100 straight-line segments. This path has the property that the creeping ant is getting continously closer and closer to B during the walk. The sum of the 100 segments in centimeters is S. What is the largest possible value of S? 40 points

19. The numbers $1, 2, ..., 8$ are paired randomly. Consider the four intervals formed by these pairs. Let p be the probability that one of these intervals intersects each of the other three.
What is the integer part of $2310 \cdot p$? 40 points

20. Alan, Bob and Colin play the following game. They start with 15, 17 and 20 dollars, respectively. In each round, two of them who still have money are randomly chosen. The chosen players then play a fair coin toss (i.e. each has a 50% chance of winning), and the loser pays the winner \$1. If someone runs out of money, he leaves. The game ends when one of the players has all the money.
What is the average number of rounds in this game? 45 points

21. Alan has written the numbers $1, 2, \ldots 81$ in the entries of a 9×9 array. Bob wants to find the position of each number in the array. He can select any square region bounded by lattice lines of the array and Alan then tells him all the numbers, in arbitrary order, contained in the selected region. At least how many questions does Bob have to put to find Alan's array? 50 points